

Hong Kong Mathematics Olympiad (1999 – 2000)

Heat Event (Individual)

香港数学竞赛 (1999 – 2000)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 设 $x = 0.\dot{1}\dot{7} + 0.0\dot{1}\dot{7} + 0.00\dot{1}\dot{7} + \dots$ ，求 x 的值。

Let $x = 0.\dot{1}\dot{7} + 0.0\dot{1}\dot{7} + 0.00\dot{1}\dot{7} + \dots$, find the value of x .

2. 解下列方程：

$$\frac{1}{x+12} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \dots + \frac{1}{(x+10)(x+11)} + \frac{1}{(x+11)(x+12)} = \frac{1}{4}.$$

Solve the following equation:

$$\frac{1}{x+12} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \dots + \frac{1}{(x+10)(x+11)} + \frac{1}{(x+11)(x+12)} = \frac{1}{4}.$$

3. 用数字 0、1、2、5 可以组成多少个能被 5 整除的三位数？（若数字不可以重复使用。）

Using digits 0, 1, 2, and 5, how many 3-digit numbers can be formed, which are divisible by 5? (If no digit may be repeated.)

4. 在图一，有一个 4×3 的矩形蜘蛛网。若有一只蜘蛛沿着网丝爬行。而其爬行方向只可向东或向北。该蜘蛛由 A 点到 C 点共有多少种可能路径？

Figure 1 represents a 4×3 rectangular spiderweb. If a spider walks along the web from A to C and it always walks either due East or due North. Find the total number of possible paths.

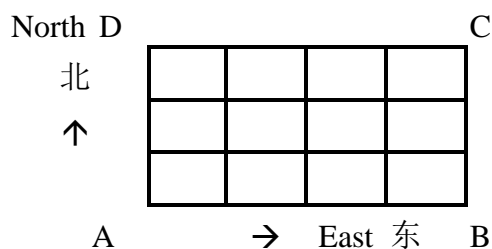


Figure 1

图一

5. 在图二，设 $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = x^\circ$ ，求 x 的值。

In Figure 2, let $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = x^\circ$, find the value of x .

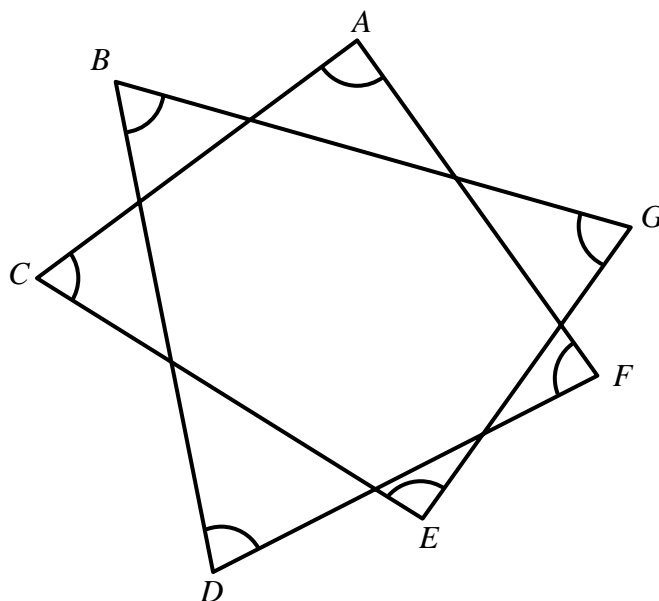


Figure 2

图二

6. 于一白纸上，画有 20 条直线。该 20 条直线，并没有两条或两条以上是平行的，也没有三条或三条以上的直线共点，问这 20 条直线最多可构成多少个交点？

Twenty straight lines were drawn on a white paper. Among them, no two or more straight lines are parallel; also no three or more than three straight lines are concurrent. What is the maximum number of intersections that these 20 lines can form?

7. 某一家庭有两个孩子，已知其中一个孩子是女的，求该家庭的另一个孩子亦是女儿的概率是多少？(假设生男、生女的概率相等。)

In a family of 2 children, given that one of them is a girl, what is the probability of having another girl? (Assuming equal probabilities of boys and girls.)

8. 有一个六位数，其个位数字为「1」，若将该个位数字「1」移至十万位，其原来的十万位数字、万位数字、千位数字、...皆向右顺移一个位。新的六位数的值为原来的六位数的值的 $\frac{1}{3}$ ，求原来的六位数。

A particular 6-digit number has a unit-digit “1”. Suppose this unit-digit “1” is moved to the place of hundred thousands, while the original ten thousand-digit, thousand-digit, hundred-digit, ...are moved one digit place to the right. The value of the new 6-digit number is one-third of the value of the original 6-digit number. Find the original 6-digit number.

9. 求 $\frac{12\sin^2 48^\circ + 12\sin^2 42^\circ}{\sin 330^\circ \tan 135^\circ - \sin^2 48^\circ \sin^2 42^\circ \tan 180^\circ}$ 的值。

Find the value of $\frac{12\sin^2 48^\circ + 12\sin^2 42^\circ}{\sin 330^\circ \tan 135^\circ - \sin^2 48^\circ \sin^2 42^\circ \tan 180^\circ}$.

10. 求直线 $3x - y - 4 = 0$ 与点 $(2, 2)$ 的最短距离。

Find the shortest distance between the line $3x - y - 4 = 0$ and the point $(2, 2)$.